A COLLECTIVE MODEL OF BARYONS

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ABSTRACT

We propose an algebraic description of the geometric structure of baryons in terms of the algebra $U(7)$. We construct a mass operator that preserves the threefold permutational symmetry and discuss a collective model of baryons with the geometry of an oblate top.

The ongoing construction of advanced accelerators in the energy range of 1–10 GeV is generating renewed interest in the spectroscopy of baryons. A detailed description of the structure of baryons requires a treatment of both the internal (spin-flavor-color) and geometric degrees of freedom. The purpose of this note is twofold: (i) to introduce a $U(7)$ algebra in order to provide a unified framework in which the geometric structure of baryons can be analyzed and (ii) to discuss, within this framework, a collective model of baryons in which the constituent parts (quarks or otherwise) move in a correlated fashion, in contrast with the single-particle picture of the quark potential model, either in its nonrelativistic [1] or relativized [2] form. The algebra $U(7)$ enlarges the scope of the algebraic models of a generation ago [3] since, within its framework, it is relatively simple to incorporate the main aspect of baryon structure, $i.e.$ its triality aspect according to which three constituent parts must be present in baryons. This leads to a threefold symmetry which has to be imposed on all operators and wave functions.

In order to introduce the $U(7)$ algebra, we begin by noting that two vector coordinates characterize the geometric structure of an object with three constituent parts. These can be conveniently taken as the Jacobi coordinates

$$
\begin{aligned}
\vec{\rho} &= (\vec{r}_1 - \vec{r}_2)/\sqrt{2} \ , \\
\vec{\lambda} &= (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)/\sqrt{6} \ ,
\end{aligned}
\tag{1}
$$

where $\vec{r}_i$ denotes the coordinate of the i-th constituent (Fig. 1). The algebraic model we propose is obtained by quantizing the Jacobi coordinates in (1) and their associated momenta. To this end, we introduce two vector bosons with angular momentum and parity, $L^\pi = 1^-$, and a scalar boson with $L^\pi = 0^+$. We denote the corresponding creation and annihilation operators by $p^\dagger_{\rho,m}$, $p^\dagger_{\lambda,m}$, $p_{\rho,m}$, $p_{\lambda,m}$ (vector bosons) and $s^\dagger$, s (scalar boson). The index m represents the three components of the vector bosons, $m = 0, \pm 1$. The $U(7)$ algebra is generated by the 49 bilinear products $G_{\alpha,\alpha'} = b^\dagger_\alpha b_{\alpha'}$, where $b^\dagger_\alpha$ ($\alpha = 1, \ldots, 7$) denotes the set of seven creation operators and $b_{\alpha'}$ the corresponding set of annihilation operators. This quantization procedure follows the general criterion of associating an algebra $U(k+1)$ to a problem with k degrees of freedom (in the present case $k = 6$) and assigning all states to the totally symmetric representation $[N]$ of $U(k+1)$. Such a bosonic quantization scheme has proved to be very useful in nuclear [4] and molecular [5] physics.

The next step is to construct the mass operator and other operators of interest in terms of elements of $U(7)$. The eigenstates of the mass operator must have well defined transformation properties under the permutation of the identical constituent parts. For nonstrange baryons, assuming charge-independence, this implies that the mass operator must be invariant under S_3. The transformation

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properties of all algebraic operators under S_3 follow from those of $s^\dagger$, $p_\rho^\dagger$ and $p_\lambda^\dagger$. There are three different symmetry classes for the permutation of three objects: a symmetric one, S, an antisymmetric one, A, and a two-dimensional class of mixed symmetry type, denoted usually by M_ρ and M_λ. The latter has the same transformation properties of the creation operators, $p_\rho^\dagger$ and $p_\lambda^\dagger$. Alternatively, the three symmetry classes can be labelled by the irreducible representations of the point group D_3 (which is isomorphic to S_3) as A_1, A_2 and E, respectively. The s-boson is a scalar under the permutation group. It is now straightforward to find bilinear combinations of bosonic operators that transform irreducibly under the permutation group [6]. We use the multiplication rules of S_3 to construct all rotationally invariant one- and two-body terms in the $U(7)$ mass operator that preserve parity and that are scalars under the permutation group

$$
\begin{aligned}
\hat{M}^2_{U(7)} = {} & \epsilon_s\, s^\dagger s - \epsilon_p\, (p_\rho^\dagger \cdot \tilde{p}_\rho + p_\lambda^\dagger \cdot \tilde{p}_\lambda) + u_0\, s^\dagger s^\dagger s s - u_1\, s^\dagger (p_\rho^\dagger \cdot \tilde{p}_\rho + p_\lambda^\dagger \cdot \tilde{p}_\lambda) s \\
& + v_0 \left[(p_\rho^\dagger \cdot p_\rho^\dagger + p_\lambda^\dagger \cdot p_\lambda^\dagger) s s + s^\dagger s^\dagger (\tilde{p}_\rho \cdot \tilde{p}_\rho + \tilde{p}_\lambda \cdot \tilde{p}_\lambda) \right] \\
& + \sum_{l=0,2} c_l \left[(p_\rho^\dagger p_\rho^\dagger - p_\lambda^\dagger p_\lambda^\dagger)^{(l)} \cdot (\tilde{p}_\rho \tilde{p}_\rho - \tilde{p}_\lambda \tilde{p}_\lambda)^{(l)} + 4\, (p_\rho^\dagger p_\lambda^\dagger)^{(l)} \cdot (\tilde{p}_\lambda \tilde{p}_\rho)^{(l)} \right] \\
& + c_1\, (p_\rho^\dagger p_\lambda^\dagger)^{(1)} \cdot (\tilde{p}_\lambda \tilde{p}_\rho)^{(1)} + \sum_{l=0,2} w_l\, (p_\rho^\dagger p_\rho^\dagger + p_\lambda^\dagger p_\lambda^\dagger)^{(l)} \cdot (\tilde{p}_\rho \tilde{p}_\rho + \tilde{p}_\lambda \tilde{p}_\lambda)^{(l)} ,
\end{aligned}
$$

$$(2)$$

with $\tilde{p}_{\lambda,m} = (-1)^{1-m} p_{\lambda,-m}$ and a similar expression for $\tilde{p}_{\rho,m}$. The parentheses (l) denote angular momentum couplings. Eq. (2) represents the most general S_3-invariant mass operator that one can construct up to quadratic terms in the elements of $U(7)$. The spectrum of this mass operator can be obtained by straightforward diagonalization in the basis provided by the symmetric irreducible representation $[N]$ of $U(7)$. The value of N determines the number of states in the model space and in view of confinement is expected to be large. For a given N the model space contains the oscillator shells with $n = n_\rho + n_\lambda = 0, 1, \ldots, N$.

The mass operator of eq. (2) spans a large class of possible geometric models of baryons. When $v_0 = 0$ the mass operator is diagonal in the harmonic oscillator basis. The linear terms correspond to an oscillator frequency of $\epsilon_p - \epsilon_s$ and the remaining terms represent anharmonic contributions. This case is the algebraic analogue of the harmonic oscillator quark model [1] and corresponds to the subalgebra $U(6) \otimes U(1)$ of $U(7)$. The v_0 term gives rise to a coupling between oscillator shells and hence generates different models of baryon structure, which are of a more collective nature (*i.e.* such that when the wave functions are expanded in an oscillator basis, they are spread over many shells). In the remaining part of this letter, we discuss one of these models, corresponding to the geometric structure of an oblate symmetric top.

In order to understand the physical content of the otherwise abstract interaction terms in eq. (2), it is convenient to analyze them in more intuitive geometric terms. Geometric shape variables can be associated with algebraic models by introducing coherent (or intrinsic) states [7]. For bosonic systems, the ground state intrinsic wave function can be written as a condensate

$$| N; c \rangle = \frac{1}{\sqrt{N!}} \left(b_c^\dagger \right)^N | 0 \rangle . \tag{3}$$

The condensate bosons of $U(7)$ are

$$b_c^\dagger = (1 + R^2)^{-1/2} \left[s^\dagger + r_\rho p_{\rho,0}^\dagger + r_\lambda \sum_m d_{m,0}^{(1)}(\theta) p_{\lambda,m}^\dagger \right] , \tag{4}$$

where $R^2 = r_\rho^2 + r_\lambda^2$. They depend on three geometric variables, two lengths r_ρ, r_λ and the relative angle θ, i.e. $\vec{r}_\rho \cdot \vec{r}_\lambda = r_\rho r_\lambda \cos \theta$. In general, the equilibrium (ground state) configuration of the condensate can be obtained by calculating the expectation value of $\hat{M}_{U(7)}^2$ in the condensate wave function and minimizing it with respect to r_ρ, r_λ and θ. The S_3-invariant operator of eq. (2) supports a variety of equilibrium configurations: spherical, linear and nonlinear. In this letter we study the rigid nonlinear shape characterized by the equilibrium values $r_\rho = r_\lambda = R/\sqrt{2} > 0$ and $\theta = \pi/2$. These are precisely the conditions satisfied by the Jacobi coordinates in eq. (1) for an equilateral triangular shape. The variables $\vec{r}_\rho$ and $\vec{r}_\lambda$ are obtained from $\vec{\rho}$ and $\vec{\lambda}$ by dividing by a scale and are therefore dimensionless.

In order to construct a mass formula corresponding to this situation, we begin by decomposing $\hat{M}_{U(7)}^2$ into a vibrational and a rotational part, $\hat{M}_{U(7)}^2 = \hat{M}_{\text{vib}}^2 + \hat{M}_{\text{rot}}^2$. The vibrational part is given by

$$\begin{aligned} \hat{M}_{\text{vib}}^2 = & \; \xi_1 \left(R^2 s^\dagger s^\dagger - p_\rho^\dagger \cdot p_\rho^\dagger - p_\lambda^\dagger \cdot p_\lambda^\dagger \right) \left(R^2 ss - \tilde{p}_\rho \cdot \tilde{p}_\rho - \tilde{p}_\lambda \cdot \tilde{p}_\lambda \right) \\ & + \xi_2 \left[\left(p_\rho^\dagger \cdot p_\rho^\dagger - p_\lambda^\dagger \cdot p_\lambda^\dagger \right) \left(\tilde{p}_\rho \cdot \tilde{p}_\rho - \tilde{p}_\lambda \cdot \tilde{p}_\lambda \right) + 4 \left(p_\rho^\dagger \cdot p_\lambda^\dagger \right) \left(\tilde{p}_\lambda \cdot \tilde{p}_\rho \right) \right] . \end{aligned} \tag{5}$$

By construction it annihilates the equilibrium condensate and depends on three parameters, ξ_1, ξ_2 and R^2, which are related to u_0, v_0, c_0 and w_0 in eq. (2).

A standard analysis [8] of the vibrational excitations gives, in the limit in which N is large, a harmonic vibrational spectrum

$$\begin{aligned} M_{\text{vib}}^2 &= N \left[\lambda_1 n_u + \lambda_2 (n_v + n_w) \right] , \\ \lambda_1 &= 4 \xi_1 R^2 , \\ \lambda_2 &= 4 \xi_2 R^2 (1 + R^2)^{-1} . \end{aligned} \tag{6}$$

The vibrations consist of a symmetric stretching (u), an antisymmetric stretching (v) and a bending vibration (w). The first two are radial excitations, whereas

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the third is an angular mode which corresponds to oscillations in the angle θ between the two Jacobi coordinates. The angular mode is degenerate with the antisymmetric radial mode. These features show that $\hat{M}^2_{\text{vib}}$ of eq. (5) describes the vibrational excitations of an oblate symmetric top.

In addition to vibrational excitations, there are rotational excitations. The rotational excitations are labelled by the orbital angular momentum, L, its projection on the threefold symmetry axis, $K = 0, 1, \ldots$, parity and the transformation properties under the permutation group. For a given value of K, the states have angular momentum $L = K, K + 1, \ldots$, and parity $\pi = (-)^K$. For rotational states built on vibrations of type A_1 (symmetric under S_3) each L state is single for $K = 0$ and twofold degenerate for $K \neq 0$. For rotational states built on vibrations of type E (mixed S_3 symmetry) each L state is twofold degenerate for $K = 0$ and fourfold degenerate for $K \neq 0$. The transformation property of the states under the permutation group is found by combining the symmetry character of the vibrational and the rotational wave functions. The rotational spectrum is obtained by returning to eq. (2) and observing that, for a S_3-invariant mass operator, there are four independent terms that determine the rotational spectrum [6]. Two of these terms, $\kappa_1 \, \hat{L} \cdot \hat{L} + \kappa_2 \, \hat{K}^2_y$, commute with any S_3-invariant operator and hence correspond to exact symmetries. Here $\hat{L} = \sqrt{2} \, (p^\dagger_\rho \tilde{p}_\rho + p^\dagger_\lambda \tilde{p}_\lambda)^{(1)}$ is the angular momentum operator and $\hat{K}_y = -i\sqrt{3} \, (p^\dagger_\rho \tilde{p}_\lambda - p^\dagger_\lambda \tilde{p}_\rho)^{(0)}$ corresponds, for large values of N, to a rotation about the threefold symmetry axis (the y-axis in our convention). The corresponding eigenvalues are $\kappa_1 \, L(L+1) + \kappa_2 \, K^2_y$ and the spectrum is shown schematically in Fig. 2. The other two rotational terms do not commute with the vibrational part of $\hat{M}^2_{U(7)}$ and thus induce rotation-vibration couplings [6]. Although within the $U(7)$ model we can take these terms into account as well, we do not consider them here.

The above indicated rotational terms do not lead to a feature of hadronic spectra expected on the basis of QCD and extensively investigated decades ago [9], namely, the occurrence of linear Regge trajectories. However, since L and $|K_y|$ are good quantum numbers, we can consider, still remaining with $U(7)$, more complicated functional forms, $f(\hat{L}^2) + g(\hat{K}^2_y)$, with eigenvalues $f[L(L+1)] + g[K^2_y]$. Linear Regge trajectories can be simply obtained by choosing the form

$$\hat{M}^2_{\text{rot}} = \alpha \sqrt{\hat{L} \cdot \hat{L} + \frac{1}{4}} \, , \tag{7}$$

with eigenvalues

$$M^2_{\text{rot}} = \alpha \, (L + 1/2) \, . \tag{8}$$

where α characterizes the slope of the trajectories.

In comparing with experimental data, the final step is to add the spin-flavor part. For this part, we take the standard $SU(6) \supset SU(3) \otimes SU(2)$ spin-flavor dynamic symmetry of Gürsey and Radicati [10]. In general, this symmetry may be broken (as it is the case with the hyperfine interaction in the quark potential model). Here we limit ourselves to the diagonal part

$$M^2_{\text{spin-flavor}} = a \left[\langle \hat{C}_{SU(6)} \rangle - 45 \right] + b \left[\langle \hat{C}_{SU(3)} \rangle - 9 \right] + c \langle \hat{C}_{SU(2)} \rangle \ . \tag{9}$$

The first term involves the Casimir operator of the $SU(6)$ spin-flavor group with eigenvalues 45, 33 and 21 for the representations $56 \leftrightarrow A_1$, $70 \leftrightarrow E$ and $20 \leftrightarrow A_2$, respectively. The second term involves the Casimir invariant of the $SU(3)$ flavor group with eigenvalues 9 and 18 for the octet and decuplet, respectively. The last term contains the eigenvalues $S(S+1)$ of the spin operator.

Combination of all pieces gives us a closed mass formula for the nonstrange baryon resonances,

$$M^2 = M_0^2 + M^2_{\text{vib}} + M^2_{\text{rot}} + M^2_{\text{spin-flavor}} \ . \tag{10}$$

This formula contains 7 parameters, M_0^2, $\lambda_1 N$, $\lambda_2 N$, α, a, b and c. We have used it to describe the spectrum of the N and Δ families of states, given in Table 1. The parameters M_0^2, b, $\lambda_1 N$ and $\lambda_2 N$ have been determined from the masses of $N(939)$, $\Delta(1232)$, $N(1440)$ and $N(1710)$, respectively. The other parameters a, c and α have been determined by fitting the masses of the remaining resonances shown in Table 1. The values of the parameters (in GeV^2) are

$$M_0^2 = 0.260 \ , \quad \lambda_1 N = 1.192 \ , \quad \lambda_2 N = 1.538 \ , \quad \alpha = 1.056 \ ,$$
$$a = -0.042 \ , \quad b = 0.029 \ , \quad c = 0.125 \ . \tag{11}$$

The value of the inverse slope of the Regge trajectories ($\alpha = 1.056$ GeV^2) is consistent with QCD estimates [9] and identical with that extracted from an analysis of meson masses [11] ($\alpha = 1.092$ GeV^2). As one can see from Table 1, the overall fit is of comparable quality to that of quark potential models [1, 2]. We have associated $N(1440)$, $\Delta(1600)$ and $\Delta(1900)$ with an A_1-vibration ($n_u = 1, n_v + n_w = 0$), and $N(1710)$ with an E-vibration ($n_u = 0, n_v + n_w = 1$). The remaining states are rotational excitations belonging to the ground band ($n_u = n_v + n_w = 0$). It is important to note that in the oblate top classification the $N(1440)$ Roper resonance is a one-phonon excitation, whereas in a harmonic oscillator quark model it is a two-phonon excitation.

In order to emphasize the difference between the model presented here and the quark potential model, it is worthwhile to compare the respective wave functions.

With the mass operator of eq. (5) the oblate top wave functions depend only on $\hat{M}^2_{\mathrm{vib}}$ (since the other terms are diagonal). In particular, the wave functions of the rotational states belonging to the ground band ($n_u = n_v + n_w = 0$) are obtained by projection from the equilibrium condensate, and therefore depend only on the value of R^2. In Figure 3, we show the expansion of the ground state wave function with $L^\tau_t = 0^+_S$ (the nucleon $N(939)$) in the harmonic oscillator basis for $R^2 = 0.2$, 1.0 and 5.0. The oblate top wave functions are spread over many oscillator shells and hence are truly collective in nature. In the harmonic oscillator limit [1] the wave function is pure $n = 0$. When perturbations are added, the nucleon wave function acquires small admixtures (19% of $n = 2$ in [12]).

Despite the large differences in wave functions, both the collective oblate top and the quark potential model give equally good descriptions of the baryon masses. This indicates that masses alone are not sufficient to distinguish between these two different scenarios. Transition form factors are far more sensitive to details in the wave functions. Preliminary study of helicity amplitudes for photocouplings ($N^* \to N + \gamma$) and the corresponding transition form factors indeed show results to this effect. These results will be presented in a subsequent publication. We also note that strange baryons can be treated with $U(7)$ in a similar fashion, except that the S_3 symmetry is lowered to S_2 when the mass of one of the constituent parts is different from the other two. Hyperfine, spin-orbit and other types of interactions can be treated as well.

In conclusion, we have introduced a unified framework for the description of the structure of baryons, which is obtained by a bosonic quantization of the Jacobi coordinates, $\vec{\rho}$ and $\vec{\lambda}$, and their associated momenta. This method extends previous algebraic treatments [13] and allows a straightforward construction of operators and wave functions with appropriate threefold permutation symmetry. The framework provides a tractable computational scheme in which all calculations can be done exactly. It encompasses both single-particle and collective forms of dynamics so that different models for the structure of baryons can be studied and compared. We have analyzed a particular collective model in which baryon resonances are treated as rotational and vibrational excitations of an oblate symmetric top. This model provides a description of baryon masses of comparable quality to that of quark potential models. Our results indicate that baryon masses are mostly determined by simple features of strong interactions, namely, the threefold permutation symmetry and the flavor-spin symmetry.

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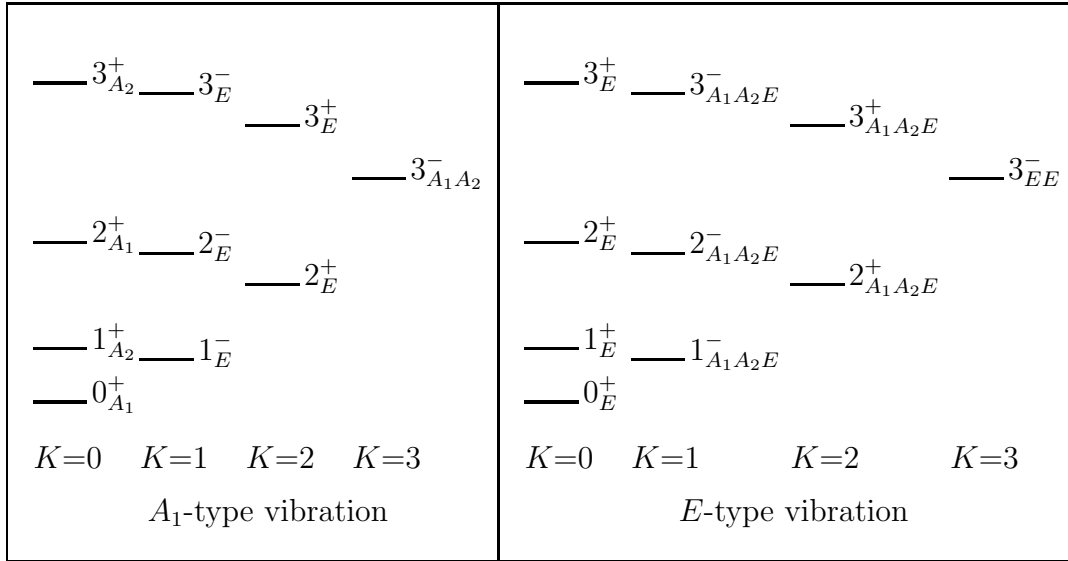
Figure 1: Geometric arrangement for baryons.

Figure 2: Schematic representation of the rotational structure built on vibrations of type A_1 (lefthand side) and vibrations of type E (righthand side). The levels are labeled by K, L_t^τ, where t denotes the overall (vibrational plus rotational) permutation symmetry. Each E state is doubly degenerate.
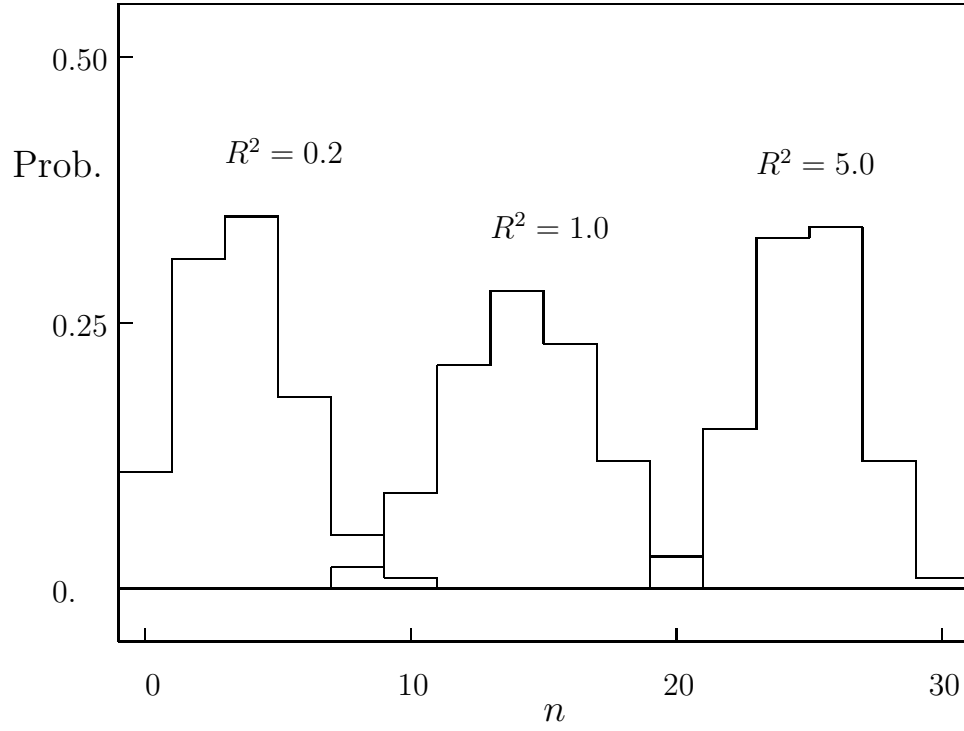
Figure 3: Probability distribution of the ground state wave function with $L_t^\pi = 0_S^+$ in a harmonic oscillator basis, calculated for $R^2 = 0.2$, 1.0 and 5.0. $n = n_\rho + n_\lambda$ is the total number of oscillator quanta. The total number of bosons is $N = 30$.

Baryon	Status	Mass	J^π	$(n_u, n_v + n_w)$	L^π, K_y	S	t	M_{calc}
$N(939)P_{11}$	****	939	$\frac{1}{2}^+$	(0,0)	$0^+, 0$	$\frac{1}{2}$	A_1	939
$N(1440)P_{11}$	****	1430-1470	$\frac{1}{2}^+$	(1,0)	$0^+, 0$	$\frac{1}{2}$	A_1	1440
$N(1520)D_{13}$	****	1515-1530	$\frac{3}{2}^-$	(0,0)	$1^-, 1$	$\frac{1}{2}$	E	1563
$N(1535)S_{11}$	****	1520-1555	$\frac{1}{2}^-$	(0,0)	$1^-, 1$	$\frac{1}{2}$	E	1563
$N(1650)S_{11}$	****	1640-1680	$\frac{1}{2}^-$	(0,0)	$1^-, 1$	$\frac{3}{2}$	E	1678
$N(1675)D_{15}$	****	1670-1685	$\frac{5}{2}^-$	(0,0)	$1^-, 1$	$\frac{3}{2}$	E	1678
$N(1680)F_{15}$	****	1675-1690	$\frac{5}{2}^+$	(0,0)	$2^+, 0$	$\frac{1}{2}$	A_1	1730
$N(1700)D_{13}$	***	1650-1750	$\frac{3}{2}^-$	(0,0)	$1^-, 1$	$\frac{3}{2}$	E	1678
$N(1710)P_{11}$	***	1680-1740	$\frac{1}{2}^+$	(0,1)	$0^+, 0$	$\frac{1}{2}$	E	1710
$N(1720)P_{13}$	****	1650-1750	$\frac{3}{2}^+$	(0,0)	$2^+, 0$	$\frac{1}{2}$	A_1	1730
$N(2190)G_{17}$	****	2100-2200	$\frac{7}{2}^-$	(0,0)	$3^-, 1$	$\frac{1}{2}$	E	2134
$N(2220)H_{19}$	****	2180-2310	$\frac{9}{2}^+$	(0,0)	$4^+, 0$	$\frac{1}{2}$	A_1	2260
$N(2250)G_{19}$	****	2170-2310	$\frac{9}{2}^-$	(0,0)	$3^-, 1$	$\frac{3}{2}$	E	2220
$N(2600)I_{1,11}$	***	2550-2750	$\frac{11}{2}^-$	(0,0)	$5^-, 1$	$\frac{1}{2}$	E	2582
$\Delta(1232)P_{33}$	****	1230-1234	$\frac{3}{2}^+$	(0,0)	$0^+, 0$	$\frac{3}{2}$	A_1	1232
$\Delta(1600)P_{33}$	***	1550-1700	$\frac{3}{2}^+$	(1,0)	$0^+, 0$	$\frac{3}{2}$	A_1	1646
$\Delta(1620)S_{31}$	****	1615-1675	$\frac{1}{2}^-$	(0,0)	$1^-, 1$	$\frac{1}{2}$	E	1644
$\Delta(1700)D_{33}$	****	1670-1770	$\frac{3}{2}^-$	(0,0)	$1^-, 1$	$\frac{1}{2}$	E	1644
$\Delta(1900)S_{31}$	***	1850-1950	$\frac{1}{2}^-$	(1,0)	$1^-, 1$	$\frac{1}{2}$	E	1974
$\Delta(1905)F_{35}$	****	1870-1920	$\frac{5}{2}^+$	(0,0)	$2^+, 0$	$\frac{3}{2}$	A_1	1905
$\Delta(1910)P_{31}$	****	1870-1920	$\frac{1}{2}^+$	(0,0)	$2^+, 0$	$\frac{3}{2}$	A_1	1905
$\Delta(1920)P_{33}$	***	1900-1970	$\frac{3}{2}^+$	(0,0)	$2^+, 0$	$\frac{3}{2}$	A_1	1905
$\Delta(1930)D_{35}$	***	1920-1970	$\frac{5}{2}^-$	(0,0)	$2^-, 1$	$\frac{1}{2}$	E	1939
$\Delta(1950)F_{37}$	****	1940-1960	$\frac{7}{2}^+$	(0,0)	$2^+, 0$	$\frac{3}{2}$	A_1	1905
$\Delta(2420)H_{3,11}$	****	2300-2500	$\frac{11}{2}^+$	(0,0)	$4^+, 0$	$\frac{3}{2}$	A_1	2396

Table 1: Oblate top classification of the nucleon and the delta families of baryon resonances. t denotes the overall permutation symmetry. The masses are given in MeV. The experimental values are taken from [14]. The average r.m.s. deviation is 38 MeV.